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Estimates for parameters and characteristics of the confining SU(3)-gluonic field in neutral kaons and chiral limit for pseudoscalar nonet

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Abstract. First part of the paper is devoted to applying the confinement mechanism proposed earlier by the author to estimate the possible parameters of the confining SU(3)-gluonic field in neutral kaons. The estimates obtained are consistent with the widths of the electromagnetic decays $K^0, \bar{K}^0 \to 2\gamma$ too. The corresponding estimates of the gluon concentrations, electric and magnetic colour field strengths are also adduced for the mentioned field at the scales of the mesons under consideration. The second part of the paper takes into account the results obtained previously by the author to estimate the purely gluonic contribution to the masses of all the mesons of pseudoscalar nonet and also to consider a possible relation with a phenomenological string-like picture of confinement. Finally, the problem of masses in particle physics is shortly discussed within the framework of approach to the chiral symmetry breaking in quantum chromodynamics (QCD) proposed recently by the author.

PACS. 12.38.-t Quantum chromodynamics – 12.38.Aw General properties of QCD (dynamics, confinement, etc.) – 14.40.Aq Pi, K, and eta mesons

1 Introduction

The present paper to some degree summarizes our previous ones on studying the pseudoscalar meson nonet within the framework of the (quark) confinement mechanism proposed earlier by the author. Global strategy may consist in reconsidering the whole spectroscopy of both mesons and baryons from the positions of the mentioned mechanism so exploring the pseudoscalar meson nonet is in essence only the first step in the given direction. But such a study involves concretizing the confinement mechanism itself in the generally accepted physical terms and when applying the mechanism to concrete hadrons there always arise new physical possibilities of interpreting the results obtained which further enriches the mechanism from physical point of view. In other words, the proposed confinement mechanism should be continuously modified and improved from physical positions and the best way of doing so is to study concrete hadrons with its help. Let us now shortly outline the main features of the approach suggested.

In [1,2,3] for the Dirac-Yang-Mills system derived from QCD-Lagrangian an unique family of compatible nonperturbative solutions was found and explored, which could pretend to decsribing confinement of two quarks. The applications of the family to the description of both the heavy quarkonia spectra [4,9] and a number of properties of pions, kaons, η- and η'-mesons [5,6,7,8,10] showed that the confinement mechanism is qualitatively the same for both light mesons and heavy quarkonia. At this moment it can be decribed in the following way.

The next main physical reasons underlie linear confinement in the mechanism under discussion. The first one is that gluon exchange between quarks is realized with the propagator different from the photon-like one, and existence and form of such a propagator is a *direct* consequence of the unique confining nonperturbative solutions of the Yang-Mills equations [2,3]. The second reason is that, owing to the structure of the mentioned propagator, quarks mainly emit and interchange the soft gluons so the gluon condensate (a classical gluon field) between quarks basically consists of soft gluons (for more details see Refs. [2, 3]) but, because of the fact that any gluon also emits gluons (still softer), the corresponding gluon concentrations rapidly become huge and form a linear confining magnetic colour field of enormous strengths, which leads to confinement of quarks. This is by virtue of the fact that just the magnetic part of the mentioned propagator is responsible for a larger portion of gluon concentrations at large distances since the magnetic part has stronger infrared singularities than the electric one. In the circumstances physically the nonlinearity of the Yang-Mills equations effectively vanishes so the latter possess the unique nonperturbative confining solutions of the Abelian-like form (with the values in Cartan subalgebra of SU(3)-Lie algebra) [2,3] which describe the gluon condensate under con-

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sideration. Moreover, since the overwhelming majority of gluons is soft they cannot leave the hadron (meson) until some gluons obtain additional energy (due to an external reason) to rush out. So we also deal with the confinement of gluons.

The approach under discussion equips us with the explicit wave functions for every two quarks (meson or quarkonium). The wave functions are parametrized by a set of real constants a_j, b_j, B_j describing the mentioned *nonperturbative* confining SU(3)-gluonic field (the gluon condensate) and they are *nonperturbative* modulo square integrable solutions of the Dirac equation in the above confining SU(3)-field and also depend on μ_0, the reduced mass of the current masses of quarks forming meson. It is clear that under the given approach just constants a_j, b_j, B_j, μ_0 determine all properties of any meson (quarkonium), i. e., the approach directly appeals to quark and gluonic degrees of freedom as should be according to the first principles of QCD. Also it is clear that the constants mentioned should be extracted from experimental data.

Such a program has been to a certain extent advanced in Refs. [4,5,6,7,8,9,10]. Under the circumstances one aim of the present paper is to complete obtaining estimates for a_j, b_j, B_j for the nonet of pseudoscalar mesons and we shall here consider neutral kaons $K^0, \bar{K}^0$. Another aim is to a certain degree to analyse some physical conclusions that can be obtained by considering the whole nonet from positions of the approach suggested.

Of course, when conducting our considerations we shall rely on the standard quark model (SQM) based on SU(3)-flavor symmetry (see, e. g., [13]) so in accordance with SQM $K^0, \bar{K}^0 = d\bar{s}, \bar{d}s$ respectively.

Section 2 contains a survey of main relations underlying description of any mesons (quarkonia) in our approach. Section 3 gives estimates for parameters of the confining SU(3)-gluonic field for neutral kaons and also contains a discussion about whether the obtained estimates might also be consistent with the widths of 2-photon decays K^0, $\bar{K}^0 \to 2\gamma$. Section 4 employs the obtained parameters of SU(3)-gluonic field to get the corresponding estimates for such characteristics of the mentioned field as gluon concentrations, electric and magnetic colour field strengths at the scales of the mesons in question while Section 5 deals with discussion about chiral limit for the nonet of pseudoscalar mesons. Section 6 takes into account the results obtained previously by the author to consider a possible relation with a phenomenological string-like picture of confinement. In section 7 the problem of masses in particle physics is shortly discussed within the framework of approach to the chiral symmetry breaking in QCD proposed recently by the author. Section 8 is devoted to the concluding remarks.

Appendices A and B contain the detailed description of main building blocks for meson wave functions in the approach under discussion, respectively: eigenspinors of the Euclidean Dirac operator on two-sphere $\mathbb{S}^2$ and radial parts for the modulo square integrable solutions of Dirac equation in the confining SU(3)-Yang-Mills field. At last, Appendix C supplements Section 2 with a proof of the uniqueness theorem from that Section in the case of SU(3)-Yang-Mills equations.

Further we shall deal with the metric of the flat Minkowski spacetime M that we write down (using the ordinary set of local spherical coordinates r, ϑ, φ for the spatial part) in the form

$$ds^2 = g_{\mu\nu} dx^\mu \otimes dx^\nu \equiv dt^2 - dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2), \quad (1)$$

so we have $|\delta| = |\det(g_{\mu\nu})| = (r^2 \sin\vartheta)^2$ and $0 \le r < \infty$, $0 \le \vartheta < \pi$, $0 \le \varphi < 2\pi$.

Throughout the paper we employ the Heaviside-Lorentz system of units with $\hbar = c = 1$, unless explicitly stated otherwise, so the gauge coupling constant g and the strong coupling constant α_s are connected by the relation $g^2/(4\pi) = \alpha_s$.

When calculating we apply the relations 1 $\mathrm{GeV}^{-1} \approx 0.1973269679$ fm , 1 $\mathrm{s}^{-1} \approx 0.658211915 \times 10^{-24}$ GeV , 1 V/m $\approx 0.2309956375 \times 10^{-23}\,\mathrm{GeV}^2$, 1 T $= 4\pi \times 10^{-7}$H/m$\times$ 1 A/m $\approx 0.6925075988 \times 10^{-15}$ GeV^2.

Finally, for the necessary estimates we shall employ the T_{00}-component (volumetric energy density) of the energy-momentum tensor for a SU(3)-Yang-Mills field which should be written in the chosen system of units in the form

$$T_{\mu\nu} = -F^a_{\mu\alpha}\, F^a_{\nu\beta}\, g^{\alpha\beta} + \frac{1}{4} F^a_{\beta\gamma}\, F^a_{\alpha\delta} g^{\alpha\beta} g^{\gamma\delta} g_{\mu\nu}\,. \quad (2)$$

2 Survey of main relations

2.1 The confining SU(3)-gluonic field and meson wave functions

As was mentioned above, our considerations shall be based on the unique family of compatible nonperturbative solutions for the Dirac-Yang-Mills system (derived from QCD-Lagrangian) studied at the whole length in Refs. [1,2,3]. Referring for more details to those references, let us briefly decribe and specify only the relations necessary to us in the present paper.

One part of the mentioned family is presented by the unique nonperturbative confining solution of the SU(3)-Yang-Mills equations for the gluonic field $A = A_\mu dx^\mu = A^a_\mu \lambda_a dx^\mu$ (λ_a are the known Gell-Mann matrices, $\mu = t, r, \vartheta, \varphi$, $a = 1, ..., 8$) and looks as follows

$$\mathcal{A}_{1t} \equiv A^3_t + \frac{1}{\sqrt{3}} A^8_t = -\frac{a_1}{r} + A_1, \mathcal{A}_{2t} \equiv -A^3_t + \frac{1}{\sqrt{3}} A^8_t = -\frac{a_2}{r} + A_2,$$

$$\mathcal{A}_{3t} \equiv -\frac{2}{\sqrt{3}} A^8_t = \frac{a_1 + a_2}{r} - (A_1 + A_2)\,,$$

$$\mathcal{A}_{1\varphi} \equiv A^3_\varphi + \frac{1}{\sqrt{3}} A^8_\varphi = b_1 r + B_1, \mathcal{A}_{2\varphi} \equiv -A^3_\varphi + \frac{1}{\sqrt{3}} A^8_\varphi = b_2 r + B_2,$$

$$\mathcal{A}_{3\varphi} \equiv -\frac{2}{\sqrt{3}} A^8_\varphi = -(b_1 + b_2) r - (B_1 + B_2) \quad (3)$$

with the real constants a_j, A_j, b_j, B_j parametrizing the family.

The word *unique* should be understood in the strict mathematical sense. In fact in Ref. [2] the following theorem was proved (see also Appendix C):

The unique exact spherically symmetric (nonperturbative) confining solutions (depending only on r and r^{-1}) of SU(3)-Yang-Mills equations in Minkowski spacetime consist of the family of (3).

It should be noted that solution (3) was found early in Ref. [1] but its uniqueness was proved just in Ref. [2] (see also Ref. [3]). Besides, in Ref. [2] (see also Ref. [5]) it was shown that the above unique confining solutions (3) satisfy the so-called Wilson confinement criterion [11]. Up to now nobody contested the above results so if we want to describe interaction between quarks by spherically symmetric SU(3)-fields then they can be only those from the above theorem. On the other hand, the desirability of spherically symmetric (colour) interaction between quarks at all distances naturally follows from analysing the $p\bar{p}$-collisions (see, e.g., Ref. [12]) where one observes a Coulomb-like potential in events which can be identified with scattering quarks on each other, i.e., actually at small distances one observes the Coulomb-like part of solution (3). Under this situation, a natural assumption will be that the quark interaction remains spherically symmetric at large distances too but then, if trying to extend the Coulomb-like part to large distances in a spherically symmetric way, we shall inevitably come to the solution (3) in virtue of the above theorem.

Now one should say that the similar unique confining solutions exist for all semisimple and non-semisimple compact Lie groups, in particular, for SU(N) with $N \geq 2$ and U(N) with $N \geq 1$ [2,3]. Explicit form of solutions, e.g., for SU(N) with $N = 2, 4$ can be found in Ref.[3] but it should be emphasized that components linear in r always represent the magnetic (colour) field in all the mentioned solutions. Especially, the case of the U(1)-group is interesting which corresponds to usual electrodynamics. Under this situation, as was pointed out in Refs. [2,3], there is an interesting possibility of indirect experimental verification of the confinement mechanism under discussion. Indeed the confining solutions of Maxwell equations for classical electrodynamics point out the confinement phase could be in electrodynamics as well. Though there exist no elementary charged particles generating a constant magnetic field linear in r, the distance from particle, after all, if it could generate this elecromagnetic field configuration in laboratory then one might study motion of the charged particles in that field. The confining properties of the mentioned field should be displayed at classical level too but the exact behaviour of particles in this field requires certain analysis of the corresponding classical equations of motion. Such a program has been recently realized in Ref. [29]. Motion of a charged (classical) particle was studied in the field representing magnetic part of the mentioned solution of Maxwell equations and it was shown that one deals with the full classical confinement of the charged particle in such a field: under any initial conditions the particle motion is accomplished within a finite region of space so that the particle trajectory is near magnetic field lines while the latter are compact manifolds (circles). Those results might be useful in thermonuclear plasma physics (for more details see [29]).

As has been repeatedly explained in Refs. [2,3,4,5], parameters $A_{1,2}$ of solution (3) are inessential for physics in question and we can consider $A_1 = A_2 = 0$. Also, as has been repeatedly discussed by us earlier (see, e. g., Refs. [2,3]), from the above form it is clear that the solution (3) is a configuration describing the electric Coulomb-like colour field (components $A^{3,8}_t$) and the magnetic colour field linear in r (components $A^{3,8}_\varphi$) and we wrote down the solution (3) in the combinations that are just needed to insert into the corresponding Dirac equation.

Another part of the family represents the meson wave functions and is given by the unique nonperturbative modulo square integrable solutions of the mentioned Dirac equation in the confining SU(3)-field of (3) $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ with the four-dimensional Dirac spinors Ψ_j representing the jth colour component of the meson, so Ψ may describe the relative motion (relativistic bound states) of two quarks in mesons and is at $j = 1, 2, 3$ (with Pauli matrix σ_1)

$$\Psi_j = e^{-i\omega_j t}\psi_j \equiv e^{-i\omega_j t} r^{-1} \begin{pmatrix} F_{j1}(r)\Phi_j(\vartheta,\varphi) \\ F_{j2}(r)\sigma_1\Phi_j(\vartheta,\varphi) \end{pmatrix}, \quad (4)$$

with the 2D eigenspinor $\Phi_j = \begin{pmatrix} \Phi_{j1} \\ \Phi_{j2} \end{pmatrix}$ of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit sphere $\mathbb{S}^2$, while the coordinate r stands for the distance between quarks.

In this situation, if a meson is composed of quarks $q_{1,2}$ with different flavours then the energy spectrum of the meson will be given by $\epsilon = m_{q_1} + m_{q_2} + \omega$ with the current quark masses m_{q_k} (rest energies) of the corresponding quarks and an interaction energy ω. On the other hand at $j = 1, 2, 3$

$$\omega_j = \omega_j(n_j, l_j, \lambda_j) =$$

$$\frac{\Lambda_j g^2 a_j b_j \pm (n_j+\alpha_j)\sqrt{(n_j^2+2n_j\alpha_j+\Lambda_j^2)\mu_0^2+g^2b_j^2(n_j^2+2n_j\alpha_j)}}{n_j^2+2n_j\alpha_j+\Lambda_j^2} \quad (5)$$

with the gauge coupling constant g while μ_0 is a mass parameter and one should consider it to be the reduced mass which is equal to $m_{q_1}m_{q_2}/(m_{q_1}+m_{q_2})$ with the current quark masses m_{q_k} (rest energies) of the corresponding quarks forming a meson (quarkonium), $a_3 = -(a_1 + a_2)$, $b_3 = -(b_1 + b_2)$, $B_3 = -(B_1 + B_2)$, $\Lambda_j = \lambda_j - gB_j$, $\alpha_j = \sqrt{\Lambda_j^2 - g^2a_j^2}$, $n_j = 0, 1, 2, ...$, while $\lambda_j = \pm(l_j + 1)$ are the eigenvalues of Euclidean Dirac operator $\mathcal{D}_0$ on a unit sphere with $l_j = 0, 1, 2, ...$

In line with the above we should have $\omega = \omega_1 = \omega_2 = \omega_3$ in energy spectrum $\epsilon = m_{q_1} + m_{q_2} + \omega$ for any meson (quarkonium) and this at once imposes two conditions on parameters a_j, b_j, B_j when choosing some experimental value for ϵ at the given current quark masses m_{q_1}, m_{q_2}.

The general form of the radial parts of (4) can be found, e.g., in Appendix B and within the given paper we need only the radial parts of (4) at $n_j = 0$ (the ground

state) that are

$$F_{j1} = C_j P_j r^{\alpha_j} e^{-\beta_j r} \left(1 - \frac{gb_j}{\beta_j}\right), P_j = gb_j + \beta_j,$$

$$F_{j2} = iC_j Q_j r^{\alpha_j} e^{-\beta_j r} \left(1 + \frac{gb_j}{\beta_j}\right), Q_j = \mu_0 - \omega_j \qquad (6)$$

with $\beta_j = \sqrt{\mu_0^2 - \omega_j^2 + g^2 b_j^2}$, while C_j is determined from the normalization condition $\int_0^\infty (|F_{j1}|^2 + |F_{j2}|^2) dr = \frac{1}{3}$. The corresponding eigenspinors of (4) with $\lambda = \pm 1$ ($l = 0$) are

$$\lambda = -1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2}, \mathrm{or} \Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ -e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2},$$

$$\lambda = 1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2}, \mathrm{or}\ \Phi = \frac{C}{2}\begin{pmatrix} -e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2} \qquad (7)$$

with the coefficient $C = 1/\sqrt{2\pi}$ (for more details, see Appendix A).

2.2 Singularities of solutions

As is seen from (3), the solutions in question have singularities: electric part contains the Coulomb-like singularities while we can rewrite the magnetic part in terms of differential 1-forms as $(b_j r + B_j)d\varphi$, j =1, 2, 3, and then pass on to Cartesian coordinates employing the relations

$$\varphi = \arctan(y/x),\ d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy$$

which entails

$$(b_j r + B_j)d\varphi = -\frac{(b_j r + B_j)y}{x^2 + y^2}dx + \frac{(b_j r + B_j)x}{x^2 + y^2}dy\,,$$

wherefrom it is obvious that the colour magnetic field of (3) has the singularities on the z-axis.

It should be noted that an analysis of singularites of YM-potentials requires both mathematical and physical considerations and, in general, is different from classical and quantum-mechanical point of view. In our recent paper [29] we gave some analysis of those singularities from classical point of view.

But let us now note the following. In classical and quantum electrodynamics (QED) it is well known (see e. g. [16]) that the notion of classical electromagnetic field (a photon condensate) generated by a charged particle is applicable only at distances much greater than the Compton wavelength $\lambda_c = 1/m$ for the given particle with mass m. Within the QCD framework the parameter Λ_{QCD} plays a similar part (see, e.g., [13]). Namely, the notion of classical SU(3)-gluonic field (a gluon condensate) is not applicable at the distances much less than $1/\Lambda_{QCD}$.

In this situation, the known singularity of the Coulomb potential in QED $\Phi = \alpha/r$ at $r = 0$ makes the purely mathematical sense since from the point of view of QED the photon condensate [huge number of (vitrual) photons] described by Φ exists only at $r >> \lambda_c$ while at $r < \lambda_c$ one may only speak about single photons rather than about condensate, i.e., the field in classical sense. The same holds true, e.g., for magnetic field of a uniformly moving charge where its strength $H \sim \mathbf{v} \times \mathbf{r}/r^3$, $\mathbf{v}$ is charge velocity.

The colour magnetic field (3) under consideration has also the singularities on the z-axis so its formal mathematical definition domain is the manifold $\mathbb{R}^3\backslash\{z\}$ with the z-axis discarded rather than the manifold $\mathbb{R}^3$. But we should not forget that we do not need only the appropriate solutions of YM-equations to describe confinement. The YM-equations are only a part of the Dirac-YM system derived from QCD-lagrangian by standard prescription. The wave functions of hadrons (at any rate, mesons) are given by the modulo square integrable solutions of the Dirac equation (which is the second important part of the above Dirac-YM system) in the field (3). But the jth colour component of wave function of two quarks (meson) (see (6)) in such a field behaves as $\psi_j \sim r^{\alpha_j} e^{-g|b_j|r}$, ($\alpha_j > 0$), at $|b_j| \to \infty$ with b_j characterising the linear colour magnetic field of solution (3), r is distance between quarks, j =1, 2, 3, $b_3 = -(b_1 + b_2)$. I.e., typical size of hadron is $r \sim 1/(g|b_j|) \to 0$ and we deal just with confinement and besides we can see that those wave functions have *no singularities* along the z-axis and are well defined there, i.e. the wave functions are well defined already on the whole $\mathbb{R}^3$. But just the wave functions are needed to calculate miscellaneous characteristics of mesons (masses, radii and so on). So it is clear that at such computations the singularities of YM-potentials in questions have no influence on physical results, i.e., those singularities are not *physical* ones.

Physically this may mean that at large distances nonlinearity of the Yang-Mills equations effectively vanishes so the latter possess the unique spherically symmetric nonperturbative confining solutions (3) (formally defined on $\mathbb{R}^3\backslash\{z\}$) of the Abelian-like form (with the values in Cartan subalgebra of SU(3)-Lie algebra) which describe the gluon condensate (a classical gluon field) leading to the confinement.

The situation is practically the same as for the hydrogen atom or positronium: the wave functions of those systems (see any textbook on quantum mechanics) are well defined at $r = 0$ so the known singularity in the Coulomb potential at $r = 0$ is also unphysical one, as said above. So modelling those singularities by some δ-functions (which is possible, as can easy show) makes no physical sense from the quantum-mechanical point of view and it can give just a suitable method for exploring some *classical* problems which is done, e.g., in many courses of classical electrodynamics while the notion of *classical chromodynamics* in fact makes no sense: we can never generate the *classical* SU(3)-Yang-Mills field and *classical coloured charged* particles at macroscopic scales.

On the other hand, just quantum considerations can lead to one more point of view on the problem of singularity along z-axis of magnetic part for solution (3) and it is presented in Section 6 of the paper.

To summarize, solutions (3) are the unique spherically-symmetric solutions of YM-equations, though mathematically being formally defined on $\mathbb{R}^3\backslash\{z\}$, but with the *unphysical* singularities on the z-axis which are inessential from quantum-mechanical point of view.

2.3 Nonrelativistic and the weak coupling limits

It is useful to specify the nonrelativistic limit (when $c \to \infty$) for the spectrum (5). For this one should replace $g \to g/\sqrt{\hbar c}$, $a_j \to a_j/\sqrt{\hbar c}$, $b_j \to b_j\sqrt{\hbar c}$, $B_j \to B_j/\sqrt{\hbar c}$ and, expanding (5) in $z = 1/c$, we shall get

$$\omega_j(n_j, l_j, \lambda_j) =$$

$$\pm\mu_0 c^2\left[1 \mp \frac{g^2 a_j^2}{2\hbar^2(n_j+|\lambda_j|)^2}z^2\right] +$$

$$\left[\frac{\lambda_j g^2 a_j b_j}{\hbar(n_j+|\lambda_j|)^2} \mp \mu_0\frac{g^3 B_j a_j^2 f(n_j,\lambda_j)}{\hbar^3(n_j+|\lambda_j|)^7}\right] z + O(z^2)\,, \quad (8)$$

where $f(n_j, \lambda_j) = 4\lambda_j n_j(n_j^2+\lambda_j^2)+\frac{|\lambda_j|}{\lambda_j}\left(n_j^4+6n_j^2\lambda_j^2+\lambda_j^4\right)$.

As is seen from (8), at $c \to \infty$ the contribution of linear magnetic colour field (parameters b_j, B_j) to the spectrum really vanishes and the

spectrum in essence becomes the purely nonrelativistic Coulomb one (modulo the rest energy). Also it is clear that when $n_j \to \infty$, $\omega_j \to \pm\sqrt{\mu_0^2 + g^2 b_j^2}$. At last, one should specify the weak coupling limit of (5), i.e., the case $g \to 0$. As is not complicated to see from (5), $\omega_j \to \pm\mu_0$ when $g \to 0$. But then quantities $\beta_j = \sqrt{\mu_0^2 - \omega_j^2 + g^2 b_j^2} \to 0$ and wave functions of (6) cease to be the modulo square integrable ones at $g = 0$, i.e., they cease to describe relativistic bound states. Accordingly, this means that the equation (5) does not make physical meaning at $g = 0$.

We may seemingly use (5) with various combinations of signes ($\pm$) before the second summand in numerators of (5) but, due to (8), it is reasonable to take all signs equal to plus which is our choice within the paper. Besides, as is not complicated to see, radial parts in the nonrelativistic limit have the behaviour of form $F_{j1}, F_{j2} \sim r^{l_j+1}$, which allows one to call quantum number l_j angular momentum for the jth colour component though angular momentum is not conserved in the field (3) [1,3]. So, for mesons under consideration we should put all $l_j = 0$.

2.4 Chiral limit

There is one more interesting limit for relation (5) – the chiral one, i.e., the situation when $m_{q1}, m_{q2} \to 0$ which entails $\mu_0 \to 0$ and (5) reduces to (at $j = 1, 2, 3$)

$$(\omega_j)_{\rm chiral} = \frac{\Lambda_j g^2 a_j b_j \pm (n_j+\alpha_j) g|b_j|\sqrt{n_j^2+2n_j\alpha_j}}{n_j^2+2n_j\alpha_j+\Lambda_j^2}\,, \quad (9)$$

which mathematically signifies that the Dirac equation in the field (3) possesses a nontrivial spectrum of bound states even for massless fermions. Physically this gives us a possible approach to the problem of chiral symmetry breaking in QCD [10]: in chirally symmetric world masses of mesons are fully determined by the confining SU(3)-gluonic field between (massless) quarks and not equal to zero. Accordingly chiral symmetry is a sufficiently rough approximation holding true only when neglecting the mentioned SU(3)-gluonic field between quarks and no additional mechanism of the spontaneous chiral symmetry breaking connected to the so-called Goldstone bosons is required. As a result, e.g., masses of mesons from pseudoscalar nonet have a purely gluonic contribution and we shall consider it in section 5.

One can note that for being the nonzero chiral limit of (5) the crucial role belongs to the colour magnetic field linear in r [parameters $b_{1,2}$ from solution (3)] inasmuch as chiral limit is equal exactly to zero when $b_{1,2} = 0$. On the contrary, when parameters $a_{1,2}$ of the Coulomb colour electric part of solution (3) are equal to zero, the chiral limit may be nonzero at $b_{1,2} \neq 0$, as is seen from (9) except for the case $n_j = 0$ when both parts of SU(3)-gluonic field (3) are important for confinement and mass generation in chiral limit.

2.5 Choice of quark masses and the gauge coupling constant

Obviously, we should choose a few quantities that are the most important from the physical point of view to characterize mesons under consideration and then we should evaluate the given quantities within the framework of our approach. In the circumstances let us settle on the ground state energy (mass) of neutral kaons, the root-mean-square radius of them and the magnetic moment. All three magnitudes are essentially nonperturbative ones, and can be calculated only by nonperturbative techniques.

Within the present paper we shall use relations (5) at $n_j = 0 = l_j$ so energy (mass) of mesons under consideration is given by $\mu = m_d + m_s + \omega$ with $\omega = \omega_j(0, 0, \lambda_j)$ for any $j = 1, 2, 3$ whereas

$$\omega = \frac{g^2 a_1 b_1}{\Lambda_1} + \frac{\alpha_1\mu_0}{|\Lambda_1|} = \frac{g^2 a_2 b_2}{\Lambda_2} + \frac{\alpha_2\mu_0}{|\Lambda_2|} =$$

$$\frac{g^2 a_3 b_3}{\Lambda_3} + \frac{\alpha_3\mu_0}{|\Lambda_3|} = \mu - m_d - m_s \quad (10)$$

and, as a consequence, the corresponding meson wave functions of (4) are represented by (6) and (7). It is evident for employing the above relations we have to assign some values to quark masses and gauge coupling constant g. We take the current quark masses used in [6,7,8,9,10] and they are $m_d = 5$ MeV, $m_s = 107.5$ MeV. Under the circumstances, the reduced mass μ_0 of (5) will be equal to $m_d m_s/(m_d + m_s)$. As to the gauge coupling constant $g = \sqrt{4\pi\alpha_s}$, it should be noted that recently some attempts have been made to generalize the standard formula for $\alpha_s = \alpha_s(Q^2) = 12\pi/[(33-2n_f)\ln(Q^2/\Lambda^2)]$ (n_f is

number of quark flavours) holding true at the momentum transfer $\sqrt{Q^2} \to \infty$ to the whole interval $0 \le \sqrt{Q^2} \le \infty$. If employing one such a generalization used in Refs. [14] which we have already discussed elsewhere (for more details see [6,7,8,9,10]) then (when fixing $\Lambda = 0.234$ GeV, $n_f = 3$) we obtain $g \approx 5.290449085$ necessary for our further computations at the mass scale of neutral kaons.

2.6 Electric form factor and the root-mean-square radius

The relations (4), (6) and (7) allow us to compute an electric formfactor of a meson as a function of the square of momentum transfer Q^2 in the form (for more details see [6,7,8,9,10])

$$f(Q^2) = \sum_{j=1}^{3} f_j(Q^2) =$$

$$\sum_{j=1}^{3} \frac{(2\beta_j)^{2\alpha_j+1}}{6\alpha_j} \cdot \frac{\sin\left[2\alpha_j \arctan\left(\sqrt{|Q^2|}/(2\beta_j)\right)\right]}{\sqrt{|Q^2|}(4\beta_j^2 - Q^2)^{\alpha_j}} \qquad (11)$$

which also entails the root-mean-square radius of the meson (quarkonium) in the form

$$< r >= \sqrt{\sum_{j=1}^{3} \frac{2\alpha_j^2 + 3\alpha_j + 1}{6\beta_j^2}} \qquad (12)$$

that is in essence a radius of confinement.

2.7 Magnetic moment

Also it is not complicated to show with the help (4), (6) and (7) that the magnetic moments of mesons (quarkonia) with the wave functions of (4) (at $l_j = 0$) are equal to zero [6,7,8,9,10], as should be according to experimental data [13].

Though we can also evaluate the magnetic form factor $F(Q^2)$ of meson (quarkonium) which is also a function of Q^2 (see Refs. [6,7]) the latter will not be used in the given paper so we shall not dwell upon it.

3 Estimates for parameters of SU(3)-gluonic field in neutral kaons

3.1 Basic equations and numerical results

Now we are able to estimate parameters a_j, b_j, B_j of the confining SU(3)-field (3) for neutral kaons within framework of our approach. In this situation, we should consider (10) and (12) the system of equations which should be solved compatibly if taking $\mu = 497.648$ MeV, $m_d = 5.0$ MeV, $m_s = 107.5$ MeV and $< r >\approx 0.560$ fm in accordance with [13]. While computing for distinctness we take all eigenvalues λ_j of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit 2-sphere $\mathbb{S}^2$ equal to 1. The results of numerical compatible solving of equations (10) and (12) are adduced in Tables 1–2.

3.2 Consistency with the widths of 2-photon decays $K^0, \bar{K}^0 \to 2\gamma$

Let us consider whether the estimates of previous subsection are consistent with the width of the electromagnetic 2-photon decays $K^0, \bar{K}^0 \to 2\gamma$. Actually kinematic analysis based on Lorentz- and gauge invariances gives rise to the following expression for the width Γ of the electromagnetic decay $P \to 2\gamma$ (where P stands for any meson from π^0, η, η', $K^0, \bar{K}^0$, see, e.g., Ref. [15])

$$\Gamma = \frac{1}{4}\pi\alpha_{em}^2 g_{P\gamma\gamma}^2 \mu^3 \qquad (13)$$

with the electromagnetic coupling constant α_{em}=1/137.0359895 and the P-meson mass μ while the information about strong interaction of quarks in P-meson is encoded in a decay constant $g_{P\gamma\gamma}$. Making replacement $g_{P\gamma\gamma} = f_P/\mu$ we can reduce (13) to the form

$$\Gamma = \frac{\pi\alpha_{em}^2 \mu f_P^2}{4}\,. \qquad (14)$$

Now it should be noted that the only invariant which f_P might depend on is $Q^2 = \mu^2$, i. e. we should find such a function $\mathcal{F}(Q^2)$ for that $\mathcal{F}(Q^2 = \mu^2) = f_P$ but $\mathcal{F}(Q^2)$ cannot be computed by perturbative techniques. It is obvious from the physical point of view that $\mathcal{F}(Q^2)$ should be connected with the electromagnetic properties of P-meson. As we have seen in Section 3, there are at least two suitable functions for this aim – electric and magnetic form factors. But there exist no experimental consequences related to a magnetic form factor at present whereas electric one to some extent determines, e. g., an effective size of meson (quarkonium) in the form $< r >$ of (12). It is reasonable, therefore, to take $\mathcal{F}(Q^2 = \mu^2) = Af(Q^2 = \mu^2)$ with some constant A and the electric form factor f of (11) for the sought relation. In this situation, we obtain an additional equation imposed on parameters of the confining SU(3)-gluonic field in P-meson which has been used in Refs. [6,7] to estimate the mentioned parameters in π^0- and η-mesons. As a result, using (11) in the case of neutral kaons, we come from (14) to relation

$$\Gamma = \frac{\pi\alpha_{em}^2 \mu}{4}\left(A\sum_{j=1}^{3}\frac{1}{6\alpha_j x_j}\cdot\frac{\sin\left(2\alpha_j \arctan x_j\right)}{(1-x_j^2)^{\alpha_j}}\right)^2 \approx$$

$$\begin{cases} 0.209\times10^{-10}\,\mathrm{eV},\ K_S^0 - \mathrm{mode}, \\ 0.696\times10^{-11}\,\mathrm{eV},\ K_L^0 - \mathrm{mode} \end{cases} \qquad (15)$$

with $x_j = \mu/(2\beta_j)$, $\mu = 497.648$ MeV and we used widths $\Gamma_7 \approx 0.209\times10^{-10}$ eV, $\Gamma_{17} \approx 0.696\times10^{-11}$ eV for decays $K^0, \bar{K}^0 \to 2\gamma$, respectively, for K_S^0 - and K_L^0-modes following the notation from Ref. [13]. In the circumstances, we can employ the results of Table 1 and compute the left-hand side of (15) which entails the corresponding values $A \approx 0.3263\times10^{-7}$ and $A \approx 0.1884\times10^{-7}$. Consequently, we draw the conclusion that parameters of the confining SU(3)-gluonic field in neutral kaons from Table 1 might be consistent with Γ_7 and Γ_{17} while smallness of constants A indicates the electromagnetic properties of neutral kaons to be inessential.

Table 1. Gauge coupling constant, reduced mass μ_0 and parameters of the confining SU(3)-gluonic field for neutral kaons

Particle	g	μ_0 (MeV)	a_1	a_2	b_1 (GeV)	b_2 (GeV)	B_1	B_2
$K^0, \bar{K}^0 - d\bar{s}, \bar{d}s$	5.29045	4.77778	0.102484	-0.198658	0.385250	-0.130208	-0.360	-0.170

Table 2. Theoretical and experimental mass and radius of neutral kaons

Particle	Theoret. μ (MeV)	Experim. μ (MeV)	Theoret. $< r >$ (fm)	Experim. $< r >$ (fm)
$K^0, \bar{K}^0 - d\bar{s}, \bar{d}s$	$\mu = m_d + m_s + \omega_j(0, 0, 1) = 497.648$	497.648	0.550510	0.560

4 Estimates of gluon concentrations, electric and magnetic colour field strengths

Now let us recall that, according to Refs. [3,5], one can confront the field (3) with the T_{00}-component (the volumetric energy density of the SU(3)-gluonic field) of the energy-momentum tensor (2) so that

$$T_{00} \equiv T_{tt} = \frac{E^2 + H^2}{2} =$$

$$\frac{1}{2}\left(\frac{a_1^2 + a_1 a_2 + a_2^2}{r^4} + \frac{b_1^2 + b_1 b_2 + b_2^2}{r^2 \sin^2\vartheta}\right) \equiv \frac{\mathcal{A}}{r^4} + \frac{\mathcal{B}}{r^2 \sin^2\vartheta} \tag{16}$$

with electric E and magnetic H colour field strengths and with real $\mathcal{A} > 0$, $\mathcal{B} > 0$. One can also introduce magnetic colour induction $B = (4\pi \times 10^{-7}\mathrm{H/m})\, H$, where H in A/m.

To estimate the gluon concentrations we can employ (16) and, taking the quantity $\omega = \Gamma$, the full decay width of a meson, for the characteristic frequency of gluons we obtain the sought characteristic concentration n in the form

$$n = \frac{T_{00}}{\Gamma}\,, \tag{17}$$

so we can rewrite (16) in the form $T_{00} = T_{00}^{\rm coul} + T_{00}^{\rm lin}$ conforming to the contributions from the Coulomb and linear parts of the solution (3). This entails the corresponding split of n from (17) as $n = n_{\rm coul} + n_{\rm lin}$.

The parameters of Table 1 were employed when computing and for simplicity we put $\sin\vartheta = 1$ in (16). There was also used the following present-day full decay widths of mesons under consideration [13]: $\Gamma = 1/\tau$ with the life times $\tau = 0.8953 \times 10^{-10}$ s (K_S^0-mode), 5.18×10^{-8} s (K_L^0-mode), respectively, whereas the Bohr radius $a_0 = 0.529177249 \cdot 10^5$ fm [13].

Table 3 contains the numerical results for $n_{\rm coul}$, $n_{\rm lin}$, n, E, H, B for the mesons under discussion.

4.1 Concluding remarks

As is seen from Table 3, at the characteristic scales of neutral kaons the gluon concentrations are huge and the corresponding fields (electric and magnetic colour ones) can be considered to be the classical ones with enormous strenghts. The part $n_{\rm coul}$ of gluon concentration n connected with the Coulomb electric colour field is decreasing faster than $n_{\rm lin}$, the part of n related to the linear magnetic colour field, and at large distances $n_{\rm lin}$ becomes dominant. It should be emphasized that in fact the gluon concentrations are much greater than the estimates given in Table 3 because the latter are the estimates for maximal possible gluon frequencies, i.e. for maximal possible gluon impulses (under the concrete situation of neutral kaons). As was mentioned in section 1, the overwhelming majority of gluons between quarks is soft, i. e., with frequencies much less than $\Gamma = 1/\tau$ with the life time τ for K_S^0 or K_L^0, so the corresponding concentrations are much greater than those in table 3. The given picture is in concordance with the one obtained in [4,5,6,7,9,10]. As a result, the confinement mechanism developed in [1,2,3] and described early in section 1 is also confirmed by the considerations of the present paper.

It should be noted, however, that our results are of a preliminary character which is readily apparent, for example, from the fact that the current quark masses (as well as the gauge coupling constant g) used in computation are known only within the certain limits, and we can expect similar limits for the magnitudes discussed in the paper so it is neccesary for further specification of the parameters for the confining SU(3)-gluonic field in neutral kaons which can be obtained, for instance, by calculating the widths of decays $K_S^0 \to \pi^+\pi^-$ or $K_L^0 \to 3\pi^0$ with the help of wave functions discussed above and in [6,10]. We hope to continue analysing the given problems elsewhere.

5 Chiral limit for pseudoscalar nonet

Having obtained estimates for neutral kaons in previous sections we can state that at the given moment we have such estimates for all the members of pseudoscalar nonet if taking into account the results of [6,7,8,10]. Under the circumstances we can return to the chiral symmetry breaking problem in QCD whose possible resolution within the framework of the above confinement mechanism has been discussed in [10]. As was mentioned in section 2, merits of case consists in that the Dirac equation in the field (3) possesses a nontrivial spectrum of bound states even for massless fermions [(see relation (9)]. As a result, mass of any meson remains nonzero in chiral limit when masses of quarks $m_q \to 0$ and meson masses will only be expressed through the parameters of the confining SU(3)-

Table 3. Gluon concentrations, electric and magnetic colour field strengths in neutral kaons

K_S^0-mode:	$r_0 =< r >= 0.550510$ fm					
r (fm)	$n_{\rm coul}$ ($\rm m^{-3}$)	$n_{\rm lin}$ ($\rm m^{-3}$)	n ($\rm m^{-3}$)	E (V/m)	H (A/m)	B (T)
$0.1r_0$	0.285346×10^{65}	0.336488×10^{63}	0.288711×10^{65}	0.957080×10^{24}	0.139807×10^{22}	0.175687×10^{16}
r_0	0.285346×10^{61}	0.336488×10^{61}	0.621834×10^{61}	0.957080×10^{22}	0.139807×10^{21}	0.175687×10^{15}
1.0	0.262079×10^{60}	0.101976×10^{61}	0.128184×10^{61}	0.290054×10^{22}	0.769654×10^{20}	0.967175×10^{14}
$10r_0$	0.285346×10^{57}	0.336488×10^{59}	0.339342×10^{59}	0.957080×10^{20}	0.139807×10^{20}	0.175687×10^{14}
a_0	0.334217×10^{41}	0.364165×10^{51}	0.364165×10^{51}	0.103580×10^{13}	0.145443×10^{16}	0.182770×10^{10}
K_L^0-mode:	$r_0 =< r >= 0.550510$ fm					
r (fm)	$n_{\rm coul}$ ($\rm m^{-3}$)	$n_{\rm lin}$ ($\rm m^{-3}$)	n ($\rm m^{-3}$)	E (V/m)	H (A/m)	B (T)
$0.1r_0$	0.165095×10^{68}	0.194684×10^{66}	0.167042×10^{68}	0.957080×10^{24}	0.139807×10^{22}	0.175687×10^{16}
r_0	0.165095×10^{64}	0.194684×10^{64}	0.359779×10^{64}	0.957080×10^{22}	0.139807×10^{21}	0.175687×10^{15}
1.0	0.151633×10^{63}	0.590013×10^{63}	0.741646×10^{63}	0.290054×10^{22}	0.769654×10^{20}	0.967175×10^{14}
$10r_0$	0.165095×10^{60}	0.194684×10^{62}	0.196335×10^{62}	0.957080×10^{20}	0.139807×10^{20}	0.175687×10^{14}
a_0	0.193370×10^{44}	0.210697×10^{54}	0.210697×10^{54}	0.103580×10^{13}	0.145443×10^{16}	0.182770×10^{10}

gluonic field of (3). This purely gluonic residual mass of meson should be interpreted as a gluonic contribution to the meson mass.

Physically this gives us a possible approach to the problem of chiral symmetry breaking in QCD [10]: in chirally symmetric world masses of mesons are fully determined by the confining SU(3)-gluonic field between (massless) quarks and are not equal to zero. Accordingly chiral symmetry is a sufficiently rough approximation holding true only when neglecting the mentioned SU(3)-gluonic field between quarks and no additional mechanism of the spontaneous chiral symmetry breaking connected to the so-called Goldstone bosons is required. Referring for more details to [10], we can here only say that, e.g., masses of mesons from pseudoscalar nonet have a purely gluonic contribution and we may be interested in what part of the meson masses is obligatory to that contribution. Therefore, let us employ the results of both [6,7,8,10] and the present paper to estimate the mentioned contribution for all the members of pseudoscalar nonet. To pass on to obtaining the sought estimates all the necessary parameters gained in [6,7,8,10] are gathered in table 4 where we took into account that in accordance with the standard quark model based on SU(3)-flavour symmetry (see, e.g., [13]) $\pi^0 = (\bar{u}u - \bar{d}d)/\sqrt{2}$ is a superposition of two quarkonia while $\eta = (2\bar{s}s - \bar{u}u - \bar{d}d)/\sqrt{6}$ and $\eta' = (\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$ are the superpositions of three quarkonia so we have, respectively, two or three sets of parameters a_j, b_j, B_j for the corresponding particles.

The current quark masses were taken with the same values as in [6,7,8,10], i.e., $m_u = 2.25$ MeV, $m_d = 5$ MeV, $m_s = 107.5$ MeV.

Now we can note that according to our approach the mass of any meson from table 4 is given by relation [cf. (10)]

$$\mu = m_{q_1} + m_{q_2} + \frac{g^2 a_1 b_1}{\lambda_1 - gB_1} + \mu_0 \frac{\sqrt{(\lambda_1 - gB_1)^2 - g^2 a_1^2}}{|\lambda_1 - gB_1|} =$$

$$m_{q_1} + m_{q_2} + \frac{g^2 a_2 b_2}{\lambda_2 - gB_2} + \mu_0 \frac{\sqrt{(\lambda_2 - gB_2)^2 - g^2 a_2^2}}{|\lambda_2 - gB_2|} =$$

$$= m_{q_1} + m_{q_2} + \frac{g^2 (a_1 + a_2)(b_1 + b_2)}{\lambda_3 + g(B_1 + B_2)} +$$

$$\mu_0 \frac{\sqrt{[\lambda_3 + g(B_1 + B_2)]^2 - g^2 (a_1 + a_2)^2}}{|\lambda_3 + g(B_1 + B_2)|},$$

$$\mu_0 = \frac{m_{q_1} m_{q_2}}{m_{q_1} + m_{q_2}}, \tag{18}$$

and the eigenvalues of the Euclidean Dirac operator on a unit sphere $\lambda_j = -1$ for π^0 and $\lambda_j = 1$ for the rest of particles.

In chiral limit $m_{q_1}, m_{q_2} \to 0$ we obtain

$$(\mu)_{chiral} \approx \frac{g^2 a_1 b_1}{\lambda_1 - gB_1} \approx \frac{g^2 a_2 b_2}{\lambda_2 - gB_2} \approx$$

$$\frac{g^2 (a_1 + a_2)(b_1 + b_2)}{\lambda_3 + g(B_1 + B_2)} \neq 0 \,. \tag{19}$$

We can see that in chiral limit the meson masses are completely determined only by the parameters a_j, b_j, B_j of SU(3)-gluonic field between quarks, i.e. by interaction between quarks, and those masses have the purely gluonic nature. So one can use the parameters g, a_j, b_j, B_j adduced in table 4 to compute $(\mu)_{chiral}$ which in fact represents the sought gluonic contribution to the meson masses. The results of computation are gathered in table 5.

One can add to the results of table 5 that we could also calculate, e.g., the root-mean-square radii (12) of particles under consideration in chiral limit which are well defined in this limit as well. For example, $< r >_{chiral} \approx$ 0.673069 fm or 0.543223 fm, accordingly, for charged pions and kaons [10]. I.e., those values only slightly differ from the present-day experimental values 0.672 fm or 0.560 fm [13]. The same remark also holds true for the decay constants for leptonic decays f_P (P stands for charged pions and kaons, see [10] for more details). So, even in chirally symmetric world, e.g., the charged pions and kaons would have nonzero masses, the root-mean-square radii and decay constants f_P for leptonic decays and all of those quantities would be determined only by SU(3)-gluonic interaction between massless quarks, i.e. they would have

Table 4. Gauge coupling constant, reduced mass μ_0 and parameters of the confining SU(3)-gluonic field for pseudoscalar nonet.

Particle	g	μ_0 (MeV)	a_1	a_2	b_1 (GeV)	b_2 (GeV)	B_1	B_2
$\pi^0 - \bar{u}u$	6.10148	1.125	-0.0434737	-0.00680835	0.0848234	0.0433136	0.01	-0.150
$\pi^0 - \bar{d}d$	6.10148	2.50	-0.0606679	0.0251427	0.0956303	0.0648174	0.1250	-0.2450
$\pi^{\pm} - u\bar{d}, \bar{u}d$	6.09131	1.55172	0.0473002	0.0118497	0.178915	-0.119290	-0.230	0.230
$K^{\pm} - u\bar{s}, \bar{u}s$	5.30121	2.20387	0.167182	-0.0557501	0.120150	0.131046	-0.900	0.290
$K^0, \bar{K}^0 - d\bar{s}, \bar{d}s$	5.29045	4.77778	0.102484	-0.198658	0.385250	-0.130208	-0.360	-0.170
$\eta - \bar{u}u$	5.14836	1.125	-0.0328122	0.179728	0.194979	0.119737	0.255	-0.010
$\eta - \bar{d}d$	5.14836	2.50	0.147640	-0.178707	0.305728	-0.119050	-0.240	-0.010
$\eta - \bar{s}s$	5.14836	53.75	-0.0141391	-0.0806779	0.252975	-0.339250	0.260	-0.310
$\eta' - \bar{u}u$	3.91476	1.125	0.218474	-0.394718	0.618419	-0.280807	-0.300	-0.200
$\eta' - \bar{d}d$	3.91476	2.50	0.351384	-0.130858	0.278983	0.285548	-0.150	0.410
$\eta' - \bar{s}s$	3.91476	53.75	0.123645	0.124633	-0.226875	0.588802	0.410	-0.160

Table 5. Theoretical, experimental and chiral meson masses

Particle	Theoretical (MeV)	Experimental (MeV)	Chiral (MeV)	Gluonic contribution (%)
$\pi^0 - \bar{u}u$	$\mu = 2m_u + \omega_j(0,0,-1) = 134.976$	134.976	129.495	96.0
$\pi^0 - \bar{d}d$	$\mu = 2m_d + \omega_j(0,0,-1) = 134.976$	134.976	122.5	90.8
$\pi^{\pm} - u\bar{d}, \bar{u}d$	$\mu = m_u + m_d + \omega_j(0,0,1) = 139.570$	139.56995	130.8	93.7
$K^{\pm} - u\bar{s}, \bar{u}s$	$\mu = m_u + m_s + \omega_j(0,0,1) = 493.677$	493.677	382.0	77.4
$K^0, \bar{K}^0 - d\bar{s}, \bar{d}s$	$\mu = m_d + m_s + \omega_j(0,0,1) = 497.648$	497.648	380.569	76.6
$\eta - \bar{u}u$	$\mu = 2m_u + \omega_j(0,0,1) = 547.51$	547.51	542.0	99.0
$\eta - \bar{d}d$	$\mu = 2m_d + \omega_j(0,0,1) = 547.51$	547.51	535.4	97.8
$\eta - \bar{s}s$	$\mu = 2m_s + \omega_j(0,0,1) = 547.51$	547.51	280.01	51.1
$\eta' - \bar{u}u$	$\mu = 2m_u + \omega_j(0,0,1) = 957.78$	957.78	952.5	99.4
$\eta' - \bar{d}d$	$\mu = 2m_d + \omega_j(0,0,1) = 957.78$	957.78	946.0	98.8
$\eta' - \bar{s}s$	$\mu = 2m_s + \omega_j(0,0,1) = 957.78$	957.78	691.5	72.2

a purely gluonic nature. Moreover, since gluons are verily relativistic particles then the most part of masses for mesons of pseudoscalar nonet is conditioned by relativistic effects, as is seen from table 5. Further discussion of the proposed chiral symmetry breaking mechanism can be found in [10].

6 A possible relation with a phenomenological string-like picture of quark confinement

6.1 The confining potential and string tension

The results adduced in section 5 allow us to shed some light on one more problem which has been touched upon in [5,10]. As is known, for a long time up to now there exists the so-called string-like picture of quark confinement but only at qualitative phenomenological level (see, e. g., Ref. [12]). Up to now, however, it is unknown how such a picture might be warranted from the point of view of QCD. Let us in short outline as our results for pseudoscalar nonet (based on and derived from QCD-Lagrangian directly) naturally lead to possible justification of the mentioned contruction. Thereto we note that one can calculate energy $\mathcal{E}$ of gluon condensate conforming to solution (3) in a volume V through relation $\mathcal{E} = \int_V T_{00} r^2 \sin\vartheta dr d\vartheta d\varphi$ with T_{00} of (16) but one should take into account that classical T_{00} has a singularity along z-axis ($\vartheta = 0, \pi$) and we have to introduce some angle ϑ_0 so $\vartheta_0 \le \vartheta \le \pi - \vartheta_0$. As well as in Ref. [2], we may consider ϑ_0 to be a parameter determining some cone $\vartheta = \vartheta_0$ so the quark emits gluons outside of the cone. Now if there are two quarks Q_1, Q_2 and each of them emits gluons outside of its own cone $\vartheta = \vartheta_{1,2}$ (see Figs. 1, 2) then we have soft gluons (as mentioned in section 1) in regions I, II and between quarks.

Accordingly, we shall have some region V with gluon condensate between quarks Q_1, Q_2 and its vertical projection is shown in Fig. 1. Another projection of V onto a plane perpendicular to the one of Fig. 1 is sketched out in Fig. 2.

Then, as is clear from Fig. 1, for distance R between quarks we have $R = R_1 \sin\vartheta_1 + R_2 \sin\vartheta_2$ and gluonic energy between quarks will be equal to

$$\mathcal{V}(R) = \int_V T_{00} r^2 \sin\vartheta dr d\vartheta d\varphi =$$

$$\int_{r_1}^{R_1}\int_{\vartheta_1}^{\pi-\vartheta_1}\int_{-\varphi_1}^{\varphi_1}\left(\frac{\mathcal{A}}{r^2}+\frac{\mathcal{B}}{\sin^2\vartheta}\right)\sin\vartheta dr d\vartheta d\varphi +$$

$$\int_{r_2}^{R_2}\int_{\vartheta_2}^{\pi-\vartheta_2}\int_{-\varphi_2}^{\varphi_2}\left(\frac{\mathcal{A}}{r^2}+\frac{\mathcal{B}}{\sin^2\vartheta}\right)\sin\vartheta dr d\vartheta d\varphi \quad (20)$$

with constants $\mathcal{A}$, $\mathcal{B}$ defined in (16).

Fig. 1. Vertical projection of region with the gluon condensate energy between quarks.

Fig. 2. Horizontal projection of region with the gluon condensate energy between quarks.

To clarify a physical meaning of the quantities $r_{1,2}$ in Figs. 1, 2, let us recall an analogy with classical electrodynamics where is well known (see, e. g., [16] and Subsection 2.2) that the notion of classical electromagnetic field (a photon condensate) generated by a charged particle is applicable only at distances much greater than the Compton wavelength $\lambda_c = 1/m$ for the given particle with mass m. Within the QCD framework the parameter Λ_{QCD} plays a similar part (see, e.g., Ref. [13,17]). Namely, the notion of classical SU(3)-gluonic field (a gluon condensate) is not applicable at the distances much less than $1/\Lambda_{QCD}$. In accordance with subsection 2.5 we took $\Lambda_{QCD} = \Lambda = 0.234$ GeV which entails $1/\Lambda \sim 0.8433$ fm so one may consider $r_{1,2} \sim 0.1 < r >$ with the root-mean-square radius $< r >$ for a meson.

Under the circumstances, performing a simple integration in (20) with employing the relations $\int d\vartheta/\sin\vartheta = \ln\tan\vartheta/2$, $\tan\vartheta/2 = \sin\vartheta/(1+\cos\vartheta) = (1-\cos\vartheta)/\sin\vartheta$, we shall without going into details (see also Ref. [2]) obtain

$$\mathcal{V}(R_1,R_2) = \mathcal{V}_0 - \sum_{i=1}^{2}\frac{4\varphi_i\mathcal{A}\cos\vartheta_i}{R_i} + \sum_{i=1}^{2} 2\varphi_i\mathcal{B}R_i\ln\frac{1+\cos\vartheta_i}{1-\cos\vartheta_i}, \tag{21}$$

where

$$\mathcal{V}_0 = \sum_{i=1}^{2}\mathcal{V}_{0i} =$$

$$\sum_{i=1}^{2}\left(\frac{4\varphi_i\mathcal{A}\cos\vartheta_i}{r_i} - 2\varphi_i\mathcal{B}r_i\ln\frac{1+\cos\vartheta_i}{1-\cos\vartheta_i}\right).$$

For the sake of simplicity let us put $R_1 = R_2$, $\vartheta_1 = \vartheta_2 = \vartheta_0$, $\varphi_1 = \varphi_2 = \varphi_0$. Then $R_1 = R_2 = R/(2\sin\vartheta_0)$ and from (21) it follows

$$\mathcal{V}(R) = \mathcal{V}_0 + \frac{a}{R} + kR \tag{22}$$

with $a = -8\varphi_0\mathcal{A}\sin 2\vartheta_0$, $k = 2\varphi_0\frac{\mathcal{B}}{\sin\vartheta_0}\ln\frac{1+\cos\vartheta_0}{1-\cos\vartheta_0}$.

We recognize the modeling confining potential in (22) which is often used when applying to meson and heavy quarkonia physics (see, e.g., [18]). We can, however, see that phenomenological parameters $a, k, \mathcal{V}_0$ of potential (22) are expressed through more fundamental parameters a_j, b_j connected with the unique exact solution (3) of Yang-Mills equations describing confinement. One can notice that the quantity k (string tension) is usually related to the so-called Regge slope $\alpha' = 1/(2\pi k)$ and in many if not all of the papers using potential approach it is accepted $k \approx 0.18$ GeV^2 (see, e. g., [18]).

6.2 Estimates of ϑ_0, φ_0 for pseudoscalar nonet

Under the situation we have the equation

$$k = 2\varphi_0\frac{\mathcal{B}}{\sin\vartheta_0}\ln\frac{1+\cos\vartheta_0}{1-\cos\vartheta_0} \approx 0.18\ \mathrm{GeV}^2 \tag{23}$$

with $\mathcal{B} = (b_1^2 + b_1b_2 + b_2^2)/2$, so let us employ (23) to estimate ϑ_0, φ_0 if using the parameters adduced in table 4 for pseudoscalar nonet and also for the ground state of toponium η_t for that we use the parametrization from [9] with the values $a_1 = 0.361253$, $a_2 = 0.339442$, $b_1 = 48.9402$ GeV, $b_2 = 76.7974$ GeV for the parameters of solution (3). Results of computations are presented in table 6.

If taking into account that only the values of ϑ_0, φ_0 between 0 and 90° are of physical meaning and, according to Figs. 1, 2, the gluon configuration between quarks will be similar to a string-like one under the condition $\vartheta_0 \to \pi/2$, $\varphi_0 \to 0$, then we can see from table 6 that the characteristic transverse sizes $D_{1,2}$ of the gluon condensate between quarks in fact tend to zero only in the case of heavy quarks, i.e., only for heavy quarks the gluon configuration between them might practically transform into a string. As a result, there arises the string-like picture of quark confinement but the latter seems to be warranted enough only for heavy quarks. It should be emphasized that string tension k of (23) is determined just by parameters $b_{1,2}$ of linear magnetic colour field from solution (3) which indirectly confirms the dominant role of the mentioned field for confinement.

We cannot, however, speak about potential $\mathcal{V}(R)$ of (22) as describing some gluon configuration between quarks. It would be possible if the mentioned potential were a solution of Yang-Mills equations directly derived from QCD-Lagrangian since, from the QCD-point of view, any gluonic field should be a solution of Yang-Mills equations (as well as any electromagnetic field is by definition always a solution of Maxwell equations).

In reality, as was shown in Refs. [2,3] (see also Appendix C), potential of form (22) cannot be a solution of the Yang-Mills equations if simultaneously $a \neq 0, k \neq 0$. Therefore, it is impossible to obtain compatible solutions of the Yang-Mills-Dirac (Pauli, Schrödinger) system when inserting potential of form (22) into Dirac (Pauli, Schrödinger) equation. So, we draw the conclusion (mentioned as far back as in Refs. [4] and elaborated more in detail in Ref. [9]) that the potential approach seems to be inconsistent: it is not based on compatible nonperturbative solutions for the Dirac-Yang-Mills system derived from QCD-Lagrangian in contrast to our confinement mechanism. Actually potential approach for heavy quarkonia has been historically modeled on positronium theory. In the latter case, however, one uses the *unique* modulo square integrable solutions of Dirac (Schrödinger) equation in the Coulomb field [condensate of huge number of (virtual)

Table 6. Angular parameters determining the gluon condensate between quarks for pseudoscalar nonet and toponium ground state

Particle	ϑ_0	φ_0
$\pi^0 - \bar{u}u$	10°	28.84°
	30°	153.6°
$\pi^0 - \bar{d}d$	10°	18.8°
	30°	100.2°
$\pi^\pm - u\bar{d}, \bar{u}d$	30°	78.63°
	45°	166.16°
$K^\pm - u\bar{s}, \bar{u}s$	45°	87.36°
	60°	171.68°
$K^0, \bar{K}^0 - d\bar{s}, \bar{d}s$	60°	70.6°
	70°	118.0°
$\eta - \bar{u}u$	45°	54.7°
	60°	107.4°
$\eta - \bar{d}d$	45°	58.0°
	60°	114.1°
$\eta - \bar{s}s$	60°	87.2°
	70°	145.8°
$\eta' - \bar{u}u$	70°	47.3°
	80°	100.6°
$\eta' - \bar{d}d$	70°	56.9°
	80°	121.1°
$\eta' - \bar{s}s$	70°	75.5°
	80°	160.8°
$\eta_t - \bar{t}t$	60°	(0.675×10^{-3})°
	80°	(0.240×10^{-2})°
	88°	(0.123×10^{-1})°

photons], i. e., one employs the *unique* compatible nonperturbative solutions of the Maxwell-Dirac (Schrödinger) system directly derived from QED-Lagrangian to describe positronium (or hydrogen atom) spectrum.

To summarize, from the point of view of our approach both potential and string-like pictures of confinement arise only as some *effective* models derived in a certain way from the more fundamental theory based on exact solution (3) of SU(3)-Yang-Mills equations. This conlusion is in concordance with the ones obtained in [5,10].

7 Problem of masses in particle physics

7.1 Preliminaries

As is known [13], the generally accepted standard model with one Higgs doublet asserts that the masses of fundamental fermions (quarks and leptons) are acquired through the Higgs mechanism so for their masses m_i we obtain (without taking mixings into account) $m_i = f_i v/\sqrt{2}$, where the vacuum Higgs condensate $v \approx 246$ GeV and i stands for quark and lepton flavours. But little is known about the coupling constants f_i and much may be elucidated only with discovering Higgs bosons. The same holds true for the gauge bosons $W^\pm, Z$ where masses $m_W = ev/(2\sin\theta_W)$, $m_Z = ev/(\sin 2\theta_W)$ with the so-called weak angle θ_W so that $\sin^2\theta_W \approx 0.23$ and e is the elementary electric charge. If taking into account that the mass of Higgs boson $m_H = \lambda v$ with a self-interaction constant λ then it is clear that masses of all the abovementioned particles are proportional to m_H, and, consequently, the discovery of Higgs boson will not completely resolve the puzzle of origin of masses in particle physics – the question will remain where the mass m_H comes from not speaking already about the nature of the above miscellaneous constants f_i and λ.

At present, to our mind, one can single out two most promising approaches to a possible resolution for the mentioned problems: technicolour theories and preon models. Under the circumstances let us shortly outline how both these directions might be estimated from the point of view of our confinement mechanism and the chiral symmetry breaking one based on the latter and discussed above and in [10].

7.2 Technicolour theories

Referring for more details concerning those models to both early references [19] and modern status of them (see, e.g., [20]) let us note the following. The main idea of acquiring masses, e.g., for $W^\pm$ and Z bosons, consists in that a new set of the so-called techniquarks is postulated at the energy scale of order 1 TeV which interact with each other through the technigluons and it makes the massless techipions exist as Goldstone bosons. The latter give

masses to $W^{\pm}$ and Z after spontaneous symmetry breaking. It should be noted, however, those massless technipions appear as a result of violating chiral symmetry connected with technicolour QCD on the analogy with chiral symmetry breaking in usual QCD. But, as we have discussed in [10] and in section 5, the hypothetical mechanism for chiral symmetry breaking in standard QCD with appearance of Goldstone bosons (pions) seems to fail because of pions can never be massless inasmuch as they have nonzero masses even in chirally symmetric world due to gluons. The same will also perfectly hold true for technipions which would always have nonzero masses due to technigluons since technicolour QCD should manifest the confinement mechanism similar to our one in usual QCD. Therefore, technicolour theories look rather doubtful from the point of view of our confinement mechanism.

7.3 Preon models

Another cardinal approach to the problem of masses is connected with the preon models (see, e.g., [21] and references therein). Under this approch quarks, leptons and gauge vector bosons are suggested to be composed of stable spin-1/2 preons, for example, existing in three flavours and being combined according to simple rules. The main theoretical objection to preon theories is the mass paradox which arises by virtue of the Heisenberg's uncertainty principle. Scattering experiments have shown [12] that quarks and leptons are point-like up to the scales of order 10^{-3} fm which corresponds to a preon mass of order 197 GeV (due to the uncertainty principle) if the preon is confined to a box of such a size, i.e. its mass will approximately be 0.4×10^5 times greater than, e.g., that of d-quark. Thus, the preon models are faced with a mass paradox: how could quarks or electrons be made of smaller particles that would have masses of many orders of magnitude greater than the fundamental fermion masses? The paradox might be resolved by the rather dubious postulate about a large binding force between preons cancelling their mass-energies. Our confinement mechanism points out the more physically acceptable way of overcoming these obstacles. If the interaction among preons is decribed by a QCD-like theory based on, e.g., SU(N)-group with $N \geq 2$ then, according to our results [2,3], such theories should also manifest confinement to generate masses decribed by relations similar to (5) and (9). This signifies that preons might possess small masses or be just massless and, as a result, mass paradox would be removed.

8 Concluding remarks

The results of present paper as well as the ones of [4,5,6,7, 8,9,10] allow one to speak about the fact that the confinement mechanism elaborated in [1,2,3] gives new possibilities for considering many old problems of hadronic (meson) physics (such as nonperturbative computation of decay constants, masses and radii of mesons, chiral symmetry breaking and so forth) from the first principles of QCD immediately appealing to the quark and gluonic degrees of freedom. This is possible because the given mechanism is based on the unique family of compatible nonperturbative solutions for the Dirac-Yang-Mills system directly derived from QCD-Lagrangian and, as a result, the approach is itself nonperturbative, relativistic from the outset, admits self-consistent nonrelativistic limit and may be employed for any meson (quarkonium). Under the circumstances the words *quark and gluonic degrees of freedom* make exact sense: gluons come forward in the form of bosonic condensate described by parameters a_j, b_j, B_j from the unique exact solution (3) of the Yang-Mills equations while quarks are represented by their current masses m_q. Though nature of the latter is not yet totally understandable (see section 7) but the confinement mechanism under discussion indicates a possible way of overcoming this puzzle - quarks might be composed from just massless preons whose interaction would be described by a gauge theory with confinement mechanism similar to that under discussion and the current quark masses might be generated along the lines discussed in sections 5 and 7.

The given paper to a great degree summarizes studying nonet of light pseudoscalar mesons realized in [5,6,7, 8,10] within the framework of our approach and we can ascertain the fact that, on the whole, this nonet can be described from the united point of view of our confinement mechanism. In line with the above, obviously, one should now pass on to vector mesons (ρ, ϕ, ω...) and also to the light scalar mesons whose nature has been controversial over 30 years [22]. As is clear from Section 2 (see also Appendices A, B), there exists a large number of relativistic bound states in the confining SU(3)-gluonic field (3) so all the mentioned mesons can probably correspond to some of those states and be described by their own sets of parameters a_j, b_j, B_j of solution (3). More important task is, however, to explore possible ways to extend the approach over baryons, in particular, over nucleons. In this situation we shall have to deal with a relativistic 3-body problem as follows from SQM. It is clear, however, that confinement mechanism under discussion (which in essence decribes the relativistic 2-body problem) should also occupy a fitting place in the 3-body constructions. We hope to develop the given direction elsewhere.

Appendix A

We here represent some results about eigenspinors of the Euclidean Dirac operator on two-sphere $\mathbb{S}^2$ employed in the main part of the paper.

When separating variables in the Dirac equation (4) there naturally arises the Euclidean Dirac operator $\mathcal{D}_0$ on the unit two-dimensional sphere $\mathbb{S}^2$ and we should know its eigenvalues with the corresponding eigenspinors. Such a problem also arises in the black hole theory while describing the so-called twisted spinors on Schwarzschild and Reissner-Nordström black holes and it was analysed in Refs. [3,23], so we can use the results obtained therein for our aims. Let us adduce the necessary relations.

The eigenvalue equation for corresponding spinors Φ may look as follows

$$\mathcal{D}_0\Phi = \lambda\Phi. \tag{A.1}$$

As was discussed in Refs. [23], the natural form of $\mathcal{D}_0$ (arising within applications) in local coordinates ϑ, φ on the unit sphere $\mathbb{S}^2$ looks as

$$\mathcal{D}_0 = -i\sigma_1\left[i\sigma_2\partial_\vartheta + i\sigma_3\frac{1}{\sin\vartheta}\left(\partial_\varphi - \frac{1}{2}\sigma_2\sigma_3\cos\vartheta\right)\right] =$$

$$\sigma_1\sigma_2\partial_\vartheta + \frac{1}{\sin\vartheta}\sigma_1\sigma_3\partial_\varphi + \frac{\cot\vartheta}{2}\sigma_1\sigma_2 \tag{A.2}$$

with the ordinary Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

so that $\sigma_1\mathcal{D}_0 = -\mathcal{D}_0\sigma_1$.

The equation (A.1) was explored in Refs. [23]. Spectrum of D_0 consists of the numbers $\lambda = \pm(l+1)$ with multiplicity $2(l+1)$ of each one, where $l = 0, 1, 2, ...$ Let us introduce the number m such that $-l \le m \le l+1$ and the corresponding number $m' = m - 1/2$ so $|m'| \le l + 1/2$. Then the conforming eigenspinors of operator $\mathcal{D}_0$ are

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \Phi_{\mp\lambda} = \frac{C}{2}\begin{pmatrix} P^k_{m'-1/2} \pm P^k_{m'1/2} \\ P^k_{m'-1/2} \mp P^k_{m'1/2} \end{pmatrix} e^{-im'\varphi} \tag{A.3}$$

with the coefficient $C = \sqrt{\frac{l+1}{2\pi}}$ and $k = l + 1/2$. These spinors form an orthonormal basis in $L^2_2(\mathbb{S}^2)$ and are subject to the normalization condition

$$\int_{\mathbb{S}^2}\Phi^\dagger\Phi d\Omega = \int\limits_0^\pi\int\limits_0^{2\pi}(|\Phi_1|^2 + |\Phi_2|^2)\sin\vartheta d\vartheta d\varphi = 1\,. \tag{A.4}$$

Further, owing to the relation $\sigma_1\mathcal{D}_0 = -\mathcal{D}_0\sigma_1$ we, obviously, have

$$\sigma_1\Phi_{\mp\lambda} = \Phi_{\pm\lambda}\,. \tag{A.5}$$

As to functions $P^k_{m'n'}(\cos\vartheta) \equiv P^k_{m',n'}(\cos\vartheta)$ then they can be chosen by miscellaneous ways, for instance, as follows (see, e. g., Ref. [24])

$$P^k_{m'n'}(\cos\vartheta) = i^{-m'-n'}\sqrt{\frac{(k-m')!(k-n')!}{(k+m')!(k+n')!}}\left(\frac{1+\cos\vartheta}{1-\cos\vartheta}\right)^{\frac{m'+n'}{2}} \times$$

$$\times\sum_{j=\max(m',n')}^{k}\frac{(k+j)!i^{2j}}{(k-j)!(j-m')!(j-n')!}\left(\frac{1-\cos\vartheta}{2}\right)^j \tag{A.6}$$

with the orthogonality relation at m', n' fixed

$$\int\limits_0^\pi P^{*k}_{m'n'}(\cos\vartheta)P^{k'}_{m'n'}(\cos\vartheta)\sin\vartheta d\vartheta = \frac{2}{2k+1}\delta_{kk'}\,. \tag{A.7}$$

It should be noted that square of $\mathcal{D}_0$ is

$$\mathcal{D}_0^2 = -\Delta_{\mathbb{S}^2}I_2 + \sigma_2\sigma_3\frac{\cos\vartheta}{\sin^2\vartheta}\partial_\varphi + \frac{1}{4\sin^2\vartheta} + \frac{1}{4}\,, \tag{A.8}$$

while laplacian on the unit sphere is

$$\Delta_{\mathbb{S}^2} = \frac{1}{\sin\vartheta}\partial_\vartheta\sin\vartheta\partial_\vartheta + \frac{1}{\sin^2\vartheta}\partial^2_\varphi = \partial^2_\vartheta + \cot\vartheta\partial_\vartheta + \frac{1}{\sin^2\vartheta}\partial^2_\varphi, \tag{A.9}$$

so the relation (A.8) is a particular case of the so-called Weitzenböck-Lichnerowicz formulas (see Refs. [25]). Then from (A.1) it follows $\mathcal{D}_0^2\Phi = \lambda^2\Phi$ and, when using the ansatz $\Phi = P(\vartheta)e^{-im'\varphi} = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} e^{-im'\varphi}$, $P_{1,2} = P_{1,2}(\vartheta)$, the equation $\mathcal{D}_0^2\Phi = \lambda^2\Phi$ turns into

$$\left(-\partial^2_\vartheta - \cot\vartheta\partial_\vartheta + \frac{m'^2 + \frac{1}{4}}{\sin^2\vartheta} + \frac{m'\cos\vartheta}{\sin^2\vartheta}\sigma_1\right)P =$$

$$\left(\lambda^2 - \frac{1}{4}\right)P\,, \tag{A.10}$$

wherefrom all the above results concerning spectrum of $\mathcal{D}_0$ can be derived [23].

When calculating the functions $P^k_{m'n'}(\cos\vartheta)$ directly, to our mind, it is the most convenient to use the integral expression [24]

$$P^k_{m'n'}(\cos\vartheta) = \frac{1}{2\pi}\sqrt{\frac{(k-m')!(k+m')!}{(k-n')!(k+n')!}}$$

$$\int_0^{2\pi}\left(e^{i\varphi/2}\cos\frac{\vartheta}{2} + ie^{-i\varphi/2}\sin\frac{\vartheta}{2}\right)^{k-n'} \times$$

$$\left(ie^{i\varphi/2}\sin\frac{\vartheta}{2} + e^{-i\varphi/2}\cos\frac{\vartheta}{2}\right)^{k+n'} e^{im'\varphi}d\varphi \tag{A.11}$$

and the symmetry relations ($z = \cos\vartheta$)

$$P^k_{m'n'}(z) = P^k_{n'm'}(z),\ P^k_{m',-n'}(z) = P^k_{-m',n'}(z),$$

$$P^k_{m'n'}(z) = P^k_{-m',-n'}(z)\,,$$

$$P^k_{m'n'}(-z) = i^{2k-2m'-2n'}P^k_{m',-n'}(z)\,. \tag{A.12}$$

In particular

$$P^k_{kk}(z) = \cos^{2k}(\vartheta/2), P^k_{k,-k}(z) = i^{2k}\sin^{2k}(\vartheta/2),$$

$$P^k_{k0}(z) = \frac{i^k\sqrt{(2k)!}}{2^k k!}\sin^k\vartheta\,,$$

$$P^k_{kn'}(z) = i^{k-n'}\sqrt{\frac{(2k)!}{(k-n')!(k+n')!}}\times$$

$$\sin^{k-n'}(\vartheta/2)\cos^{k+n'}(\vartheta/2)\,. \tag{A.13}$$

Eigenspinors with $\lambda = \pm 1, \pm 2$

If $\lambda = \pm(l+1) = \pm 1$ then $l = 0$ and from (A.3) it follows that $k = l + 1/2 = 1/2$, $|m'| \leq 1/2$ and we need the functions $P^{1/2}_{m',\pm 1/2}$ that are easily evaluated with the help of (A.11)–(A.13) so the eigenspinors for $\lambda = -1$ are

$$\Phi = \frac{C}{2}\begin{pmatrix} \cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} - i\sin\frac{\vartheta}{2} \end{pmatrix} e^{i\varphi/2},$$

$$\Phi = \frac{C}{2}\begin{pmatrix} \cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \\ -\cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \end{pmatrix} e^{-i\varphi/2}, \tag{A.14}$$

while for $\lambda = 1$ the conforming spinors are

$$\Phi = \frac{C}{2}\begin{pmatrix} \cos\frac{\vartheta}{2} - i\sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \end{pmatrix} e^{i\varphi/2},$$

$$\Phi = \frac{C}{2}\begin{pmatrix} -\cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2} \end{pmatrix} e^{-i\varphi/2} \tag{A.15}$$

with the coefficient $C = \sqrt{1/(2\pi)}$.

It is clear that (A.14)–(A.15) can be rewritten in the form

$$\lambda = -1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2},$$

or

$$\Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ -e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2},$$

$$\lambda = 1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2},$$

or

$$\Phi = \frac{C}{2}\begin{pmatrix} -e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2}, \tag{A.16}$$

so the relation (A.5) is easily verified at $\lambda = \pm 1$.

In studying vector mesons and excited states of heavy quarkonia eigenspinors with $\lambda = \pm 2$ may also be useful. Then $k = l + 1/2 = 3/2$, $|m'| \leq 3/2$ and we need the functions $P^{3/2}_{m',\pm 1/2}$ that can be evaluated with the help of (A.11)–(A.13). Computation gives rise to

$$P^{3/2}_{3/2,-1/2} = -\frac{\sqrt{3}}{2}\sin\vartheta\sin\frac{\vartheta}{2} = P^{3/2}_{-3/2,1/2},$$

$$P^{3/2}_{3/2,1/2} = i\frac{\sqrt{3}}{2}\sin\vartheta\cos\frac{\vartheta}{2} = P^{3/2}_{-3/2,-1/2},$$

$$P^{3/2}_{1/2,-1/2} = -\frac{i}{4}\left(\sin\frac{\vartheta}{2} - 3\sin\frac{3}{2}\vartheta\right) = P^{3/2}_{-1/2,1/2},$$

$$P^{3/2}_{1/2,1/2} = \frac{1}{4}\left(\cos\frac{\vartheta}{2} + 3\cos\frac{3}{2}\vartheta\right) = P^{3/2}_{-1/2,-1/2}, \tag{A.17}$$

and according to (A.3) this entails eigenspinors with $\lambda = 2$ in the form

$$\frac{C}{2}i\frac{\sqrt{3}}{2}\sin\vartheta\begin{pmatrix} e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{i3\varphi/2},\ \frac{C}{8}\begin{pmatrix} 3e^{-i\frac{3\vartheta}{2}} + e^{i\frac{\vartheta}{2}} \\ 3e^{i\frac{3\vartheta}{2}} + e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2},$$

$$\frac{C}{8}\begin{pmatrix} -3e^{-i\frac{3\vartheta}{2}} - e^{i\frac{\vartheta}{2}} \\ 3e^{i\frac{3\vartheta}{2}} + e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2},\ \frac{C}{2}i\frac{\sqrt{3}}{2}\sin\vartheta\begin{pmatrix} -e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{-i3\varphi/2} \tag{A.18}$$

with $C = 1/\sqrt{\pi}$, while eigenspinors with $\lambda = -2$ are obtained in accordance with relation (A.5).

Appendix B

We here adduce the explicit form for the radial parts of meson wave functions from (6). At $n_j = 0$ they are given by

$$F_{j1} = C_j P_j r^{\alpha_j} e^{-\beta_j r}\left(1 - \frac{Y_j}{Z_j}\right),$$

$$F_{j2} = iC_j Q_j r^{\alpha_j} e^{-\beta_j r}\left(1 + \frac{Y_j}{Z_j}\right), \tag{B.1}$$

while at $n_j > 0$ they are given by

$$F_{j1} = C_j P_j r^{\alpha_j} e^{-\beta_j r}\times$$

$$\left[\left(1 - \frac{Y_j}{Z_j}\right) L^{2\alpha_j}_{n_j}(r_j) + \frac{P_j Q_j}{Z_j} r_j L^{2\alpha_j+1}_{n_j-1}(r_j)\right],$$

$$F_{j2} = iC_j Q_j r^{\alpha_j} e^{-\beta_j r}\times$$

$$\left[\left(1 + \frac{Y_j}{Z_j}\right) L^{2\alpha_j}_{n_j}(r_j) - \frac{P_j Q_j}{Z_j} r_j L^{2\alpha_j+1}_{n_j-1}(r_j)\right] \tag{B.2}$$

with the Laguerre polynomials $L^{\rho}_{n}(r_j)$, $r_j = 2\beta_j r$, $\beta_j = \sqrt{\mu_0^2 - \omega_j^2 + g^2 b_j^2}$ at $j = 1,2,3$ with $b_3 = -(b_1 + b_2)$, $P_j = gb_j + \beta_j$, $Q_j = \mu_0 - \omega_j$, $Y_j = P_j Q_j \alpha_j + (P_j^2 - Q_j^2) g a_j/2$, $Z_j = P_j Q_j \Lambda_j + (P_j^2 + Q_j^2) g a_j/2$ with $a_3 = -(a_1 + a_2)$, $\Lambda_j = \lambda_j - gB_j$ with $B_3 = -(B_1 + B_2)$, $\alpha_j = \sqrt{\Lambda_j^2 - g^2 a_j^2}$, while $\lambda_j = \pm(l_j + 1)$ are the eigenvalues of Euclidean Dirac operator $\mathcal{D}_0$ on unit two-sphere with $l_j = 0, 1, 2, ...$ (see Appendix A) and quantum numbers $n_j = 0, 1, 2, ...$ are defined by the relations

$$n_j = \frac{gb_j Z_j - \beta_j Y_j}{\beta_j P_j Q_j}, \tag{B.3}$$

which entails the spectrum (5). Further, C_j of (B.1)–(B.2) should be determined from the normalization condition

$$\int_0^\infty (|F_{j1}|^2 + |F_{j2}|^2) dr = \frac{1}{3}. \tag{B.4}$$

As a consequence, we shall gain that in (4) $\Psi_j \in L_2^4(\mathbb{R}^3)$ at any $t \in \mathbb{R}$ and, accordingly, $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ may describe relativistic bound states in the field (3) with the energy spectrum (5). As is clear from (B.3) at $n_j = 0$ we have $gb_j/\beta_j = Y_j/Z_j$ so the radial parts of (B.1) can be rewritten as

$$F_{j1} = C_j P_j r^{\alpha_j} e^{-\beta_j r}\left(1 - \frac{gb_j}{\beta_j}\right),$$

$$F_{j2} = iC_j Q_j r^{\alpha_j} e^{-\beta_j r}\left(1 + \frac{gb_j}{\beta_j}\right). \tag{B.5}$$

More details can be found in Refs. [1,3].

Appendix C

The facts adduced here have been obained in Refs. [2, 3] and we concisely give them only for completeness of discussion in Section 2.

To specify the question, let us note that in general the Yang-Mills equations on a manifold M can be written as

$$d*F=g(*F\wedge A-A\wedge *F)\,, \tag{C.1}$$

where a gluonic field $A=A_\mu dx^\mu=A^a_\mu\lambda_a dx^\mu$ [λ_a are the known Gell-Mann matrices, $\mu=t,r,\vartheta,\varphi$ (in the case of spherical coordinates), $a=1,...,8$], the curvature matrix (field strentgh) $F=dA+gA\wedge A=F^a_{\mu\nu}\lambda_a dx^\mu\wedge dx^\nu$ with exterior differential d and the Cartan's (exterior) product $\wedge$, while $*$ means the Hodge star operator conforming to a metric on manifold under consideration, g is a gauge coupling constant.

The most important case of M is Minkowski spacetime and we are interested in the confining solutions A of the SU(3)-Yang-Mills equations. The confining solutions were defined in Ref. [1] as the spherically symmetric solutions of the Yang-Mills equations (1) containing only the components of the SU(3)-field which are Coulomb-like or linear in r. Additionally we impose the Lorentz condition on the sought solutions. The latter condition is necessary for quantizing the gauge fields consistently within the framework of perturbation theory (see, e. g. Ref. [26]), so we should impose the given condition that can be written in the form $\mathrm{div}(A)=0$, where the divergence of the Lie algebra valued 1-form $A=A_\mu dx^\mu=A^a_\mu\lambda_a dx^\mu$ is defined by the relation (see, e. g., Refs. [27])

$$\mathrm{div}(A)=\frac{1}{\sqrt{\delta}}\partial_\mu(\sqrt{\delta}g^{\mu\nu}A_\nu)\,. \tag{C.2}$$

It should be emphasized that, from the physical point of view, the Lorentz condition reflects the fact of transversality for gluons that arise as quanta of SU(3)-Yang-Mills field when quantizing the latter (see, e. g., Ref. [26]).

We shall use the Hodge star operator action on the basis differential 2-forms on Minkowski spacetime with local coordinates t,r,ϑ,φ in the form

$$*(dt\wedge dr)=-r^2\sin\vartheta d\vartheta\wedge d\varphi\,, *(dt\wedge d\vartheta)=\sin\vartheta dr\wedge d\varphi\,,$$

$$*(dt\wedge d\varphi)=-\frac{1}{\sin\vartheta}dr\wedge d\vartheta\,, *(dr\wedge d\vartheta)=\sin\vartheta dt\wedge d\varphi\,,$$

$$*(dr\wedge d\varphi)=-\frac{1}{\sin\vartheta}dt\wedge d\vartheta\,, *(d\vartheta\wedge d\varphi)=\frac{1}{r^2\sin\vartheta}dt\wedge dr\,, \tag{C.3}$$

so that on 2-forms $*^2=-1$. More details about the Hodge star operator can be found in [27].

The most general ansatz for a spherically symmetric solution is $A=A_t(r)dt+A_r(r)dr+A_\vartheta(r)d\vartheta+A_\varphi(r)d\varphi$. But then the Lorentz condition (C.2) for the given ansatz gives rise to

$$\sin\vartheta\partial_r(r^2A_r)+\partial_\vartheta(\sin\vartheta A_\vartheta)=0,$$

which yields $A_r=\frac{C}{r^2}-\frac{\cot\vartheta}{r^2}\int A_\vartheta(r)dr$ with a constant matrix C. But the confining solutions should be spherically symmetric and contain only the components which are Coulomb-like or linear in r, so one should put $C=A_\vartheta(r)=0$. Consequently, the ansatz $A=A_t(r)dt+A_\varphi(r)d\varphi$ is the most general spherically symmetric one.

For the latter ansatz we have $F=dA+gA\wedge A=-\partial_rA_tdt\wedge dr+\partial_rA_\varphi dr\wedge d\varphi+g[A_t,A_\varphi]dt\wedge d\varphi$, where $[\cdot,\cdot]$ signifies matrix commutator.

Then, according to (C.3), we obtain

$$*F=(r^2\sin\vartheta)\partial_rA_td\vartheta\wedge d\varphi-\frac{1}{\sin\vartheta}\partial_rA_\varphi dt\wedge d\vartheta-$$
$$\frac{g}{\sin\vartheta}[A_t,A_\varphi]dr\wedge d\vartheta\,, \tag{C.4}$$

which entails

$$d*F=\sin\vartheta\partial_r(r^2\partial_rA_t)\,dr\wedge d\vartheta\wedge d\varphi+\frac{1}{\sin\vartheta}\partial^2_rA_\varphi\,dt\wedge dr\wedge d\vartheta, \tag{C.5}$$

while

$$*F\wedge A-A\wedge *F=\left(r^2\sin\vartheta[\partial_rA_t,A_t]-\frac{1}{\sin\vartheta}[\partial_rA_\varphi,A_\varphi]\right)dt\wedge d\vartheta\wedge d\varphi$$
$$-\frac{g}{\sin\vartheta}\left([[A_t,A_\varphi],A_t]\,dt\wedge dr\wedge d\vartheta+[[A_t,A_\varphi],A_\varphi]\,dr\wedge d\vartheta\wedge d\varphi\right). \tag{C.6}$$

Under the circumstances the Yang-Mills equations (C.1) are tantamount to the conditions

$$\partial_r(r^2\partial_rA_t)=-\frac{g^2}{\sin^2\vartheta}[[A_t,A_\varphi],A_\varphi], \tag{C.7}$$

$$\partial^2_rA_\varphi=-g^2[[A_t,A_\varphi],A_t], \tag{C.8}$$

$$r^2\sin\vartheta[\partial_rA_t,A_t]-\frac{1}{\sin\vartheta}[\partial_rA_\varphi,A_\varphi]=0. \tag{C.9}$$

The key equation is (C.7) because the matrices A_t,A_φ depend on merely r and (C.7) can be satisfied only if the matrices $A_t=A^a_t\lambda_a$ and $A_\varphi=A^a_\varphi\lambda_a$ belong to the so-called Cartan subalgebra of the SU(3)-Lie algebra. Let us remind that, by definition, a Cartan subalgebra is a maximal abelian subalgebra in the corresponding Lie algebra, i. e., the commutator for any two matrices of the Cartan subalgebra is equal to zero (see, e.g., Ref. [28]). For SU(3)-Lie algebra the conforming Cartan subalgebra is generated by the Gell-Mann matrices λ_3,λ_8 which are

$$\lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}. \tag{C.10}$$

Under the situation we should have $A_t=A^3_t\lambda_3+A^8_t\lambda_8$ and $A_\varphi=A^3_\varphi\lambda_3+A^8_\varphi\lambda_8$, then $[A_t,A_\varphi]=0$ and we obtain

$$\partial_r(r^2\partial_rA_t)=0,\partial^2_rA_\varphi=0, \tag{C.11}$$

while (C.9) is identically satisfied and (C.11) gives rise to the solution (3) with real constants a_j,A_j,b_j,B_j parametrizing the solution which proves the uniqueness theorem of Section 2 for the SU(3) Yang-Mills equations.

The more explicit form of (3) is

$$A_t^3 = [(a_2 - a_1)/r + A_1 - A_2]/2,$$

$$A_t^8 = [A_1 + A_2 - (a_1 + a_2)/r]\sqrt{3}/2\,,$$

$$A_\varphi^3 = [(b_1 - b_2)r + B_1 - B_2]/2,$$

$$A_\varphi^8 = [(b_1 + b_2)r + B_1 + B_2]\sqrt{3}/2\,. \quad \text{(C.12)}$$

Clearly, the obtained results may be extended over all SU(N)-groups with $N \geq 2$ and even over all semisimple compact Lie groups since for them the corresponding Lie algebras possess just the only Cartan subalgebra. Also we can talk about the compact non-semisimple groups, for example, U(N). In the latter case additionally to Cartan subalgebra we have centrum consisting from the matrices of the form αI_N (I_N is the unit matrix $N \times N$) with arbitrary constant α.

The most relevant physical cases are of course U(1)- and SU(3)-ones (QED and QCD). In particular, the U(1)-case allows us to build the classical model of confinement (see Section 2 and Ref. [29]).

At last, it should also be noted that the nontrivial confining solutions obtained exist at any gauge coupling constant g, i. e. they are essentially *nonperturbative* ones.

References

1. Yu. P. Goncharov, Mod. Phys. Lett. **A 16**, 557 (2001)
2. Yu. P. Goncharov, Phys. Lett. **B 617**, 67 (2005)
3. Yu. P. Goncharov, in: P. V. Kreitler (Ed.), New Developments in Black Hole Research, Nova Science Publishers, New York, 2006, pp. 67–121 (Chapter 3), hep-th/0512099.
4. Yu. P. Goncharov, Europhys. Lett. **62**, 684 (2003);
   Yu. P. Goncharov, E. A. Choban, Mod. Phys. Lett. **A 18**, 1661 (2003);
   Yu. P. Goncharov, A. A. Bytsenko, Phys. Lett. **B 602**, 86 (2004)
5. Yu. P. Goncharov, Phys. Lett. **B 641**, 237 (2006)
6. Yu. P. Goncharov, Phys. Lett. **B 652**, 310 (2007)
7. Yu. P. Goncharov, Mod. Phys. Lett. **A 22**, 2273 (2007)
8. Yu. P. Goncharov, J. Phys. G: Nucl. Part. Phys. **35**, 095006 (2008)
9. Yu. P. Goncharov, Nucl. Phys. **A 808**, 73 (2008)
10. Yu. P. Goncharov, Nucl. Phys. **A 812**, 99 (2008)
11. K. Wilson, Phys. Rev. **D 10**, 2445 (1974); M. Bander, Phys. Rep. **75**, 205 (1981).
12. D. H. Perkins, *Introduction to High Energy Physics* (Cambridge University Press, Cambridge, 2000).
13. C. Amsler, et al., Particle Data Group, Phys. Lett. **B 667**, 1 (2008)
14. A. Deur, Nucl. Phys. **A 755**, 353 (2005);
    A. Deur, et. al., Phys. Lett. **B 650**, 244 (2007)
15. S.M. Bilenky, *Introduction to the Feynman Diagram Technique* (Atomizdat, Moscow, 1971);
    H.M. Pilkuhn, *Relativistic Particle Physics* (Springer, Berlin, 1979).
16. L. D. Landau, E. M. Lifshits, *Field Theory* (Nauka, Moscow, 1988).
17. K. Huang, *Quarks, Leptons and Gauge Fields* (World-Scientific, Singapore, 1982);
    F. J. Yndurain, *Quantum Chromodynamics. An Introduction to the Theory of Quarks and Gluons* (Springer-Verlag, Berlin, 1983).
18. N. Brambilla, et al., *CERN Yellow Report, CERN-2005-005* Geneva: CERN, 2005, hep-ph/0412158.
19. E. Farhi, L. Susskind, Phys. Rep. **74**, 277 (1981);
    R. K. Kaul, Rev. Mod. Phys. **55**, (1983) 449;
    T.-P. Cheng, L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Clarendon Press, Oxford, 1984).
20. J. Hirn, A. Martin, V. Sanz, Phys. Rev. **D 78**, 075026 (2008)
21. J. -J. Dugne, S. Fredriksson, J. Hansson, Europhys. Lett. **57**, 188 (2002)
22. F. E. Close, N. A. Törnqvist, J. Phys. G: Nucl. Part. Phys. **28**, R249 (2002)
23. Yu. P. Goncharov, Pis'ma v ZhETF **69**, 619 (1999);
    Yu. P. Goncharov, Phys. Lett. **B 458**, 29 (1999)
24. N. Ya. Vilenkin, *Special Functions and Theory of Group Representations* (Nauka, Moscow, 1991).
25. *Géométrie Riemannian en Dimension 4. Seminaire Arthur Besse* (Cedic/Fernand Nathan, Paris, 1981).
26. L. H. Ryder, *Quantum Field Theory* (Cambridge Univ. Press, Cambridge, 1985).
27. A. L. Besse, *Einstein Manifolds* (Springer-Verlag, Berlin, 1987);
    H. B. Lawson, Jr., M.-L. Michelsohn, *Spin Geometry* (Princeton University Press, Princeton, 1989);
    M. M. Postnikov, *Riemannian Geometry* (Factorial, Moscow, 1998).
28. A. O. Barut, R. Raczka, *Theory of Group Representations and Applications* (Polish Science, Warszawa, 1977).
29. Yu. P. Goncharov, N. E. Firsova, Int. J. Theor. Phys. **49**, 1155 (2010)